Fundamental Limits on Conversion Loss of Double Sideband Resistive Mixers

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Abstract—**Although the resistive mixer has been the subject of numerous studies [1]–[3], these have all dealt with specific cases for terminations at the higher order mixing products (idlers). This paper deals with the general case of the double sideband mixer, and demonstrates that when no energy is dissipated at the idler frequencies the fundamental limit on conversion loss is 3 dB, with the lost energy being equally divided between conversion to the image and reflection loss at the signal port. Also treated is the case where matched loads are presented to each idler. It is shown that, in this case, the theoretical limit on conversion loss is 3.92 dB (20 log $\pi/2$), independent of the mixer configuration.**

Introduction

THE RESISTIVE MIXER comprises one or more diodes pumped by a local oscillator (LO). The first-order analysis assumes that the signal level is significantly lower than that of the LO, and does not, therefore, perturb the LO-pumped diode conductance waveform. Under this assumption, no harmonics of the signal are generated, and the mixing products are defined by

$$v_{\mathrm{OUT}} = v_{\mathrm{SIG}} \cdot \sum_{n=1}^{\infty} [k_n \cos(n\omega_{\mathrm{LO}} - \omega_{\mathrm{SIG}})t + k_n \cos(n\omega_{\mathrm{LO}} + \omega_{\mathrm{SIG}})t]. \quad (1)$$

Fig. 1 shows the spectral distribution for a resistive mixer. The key frequencies are

$$\omega_{\mathrm{IF}} = \omega_{\mathrm{LO}} - \omega_{\mathrm{SIG}} \quad (2)$$

$$\omega_{\mathrm{IMAGE}} = 2\omega_{\mathrm{LO}} - \omega_{\mathrm{SIG}}. \quad (3)$$

The remaining mixing products (idlers) are grouped in pairs about the harmonics of the LO frequency.

Double Sideband Mixers

The double sideband mixer is one in which an IF output is generated for a signal above or below the LO. Each is the "image" of the other. Double sideband mixers are utilized in radiometers. They are also utilized as a basic building block in single sideband receivers.

Fig. 2 is a schematic representation of a double sideband mixer. Although a single physical port serves as the signal and image terminals, they are shown as two separate ports for mathematical analysis. The signal port is designated as port 1; the image port, port 2; the IF port, port 3.

Manuscript received March 29, 1976; revised November 2, 1976.

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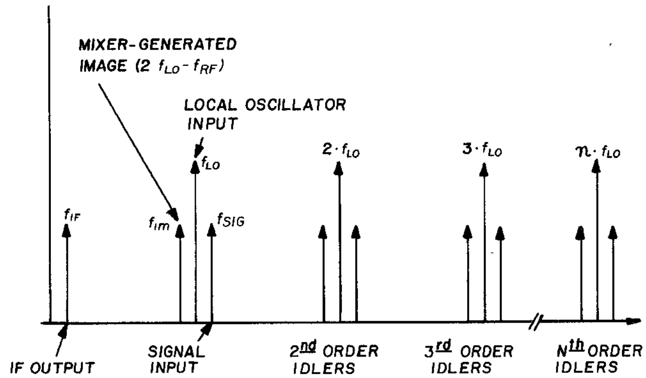

Fig. 1. Frequency spectrum of a resistive mixer excited by a local oscillator and signal.

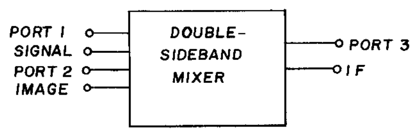

Fig. 2. Three-port equivalent circuit of a double-sideband mixer.

Fundamental Limit on Conversion Loss

The most efficient double sideband mixer is one in which real power flow is allowed to take place only at the signal, image, and intermediate frequencies. For this to occur, the idlers must be reactively terminated. To maintain generality, the nature of these terminations is left unspecified in this analysis.

To establish the fundamental limit on conversion loss, a "perfect" diode is assumed. By "perfect," it is meant that the diode has no parasitics and is driven between two states—perfect open circuit and perfect short circuit. With these assumptions, and the fact that the idlers are reactively terminated, the mixer reduces to a lossless three port, where port 1 is the signal, port 2 is the image, and port 3 is the IF:

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}. \quad (4)$$

Since the mixer and image ports are physically one and the same, the matrix can be rewritten as

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{11} & S_{13} \\ S_{31} & S_{31} & S_{33} \end{bmatrix}. \quad (5)$$

Since the mixer is lossless, the following matrix equation holds (unnormalized s parameters are used):

$$\tilde{S}^* Y_0 S = Y_0 \quad (6)$$

where

$$Y_0 = \begin{bmatrix} Y_{01} & 0 & 0 \\ 0 & Y_{01} & 0 \\ 0 & 0 & Y_{03} \end{bmatrix}. \tag{7}$$

Performing the matrix multiplication shown in (6) yields four independent equations:

$$Y_{01}|S_{11}|^2 + Y_{01}|S_{21}|^2 + Y_{03}|S_{31}|^2 = Y_{01} \tag{8}$$

$$Y_{01}S_{11}^*S_{12} + Y_{01}S_{21}^*S_{11} + Y_{03}|S_{31}|^2 = 0 \tag{9}$$

$$Y_{01}S_{11}^*S_{13} + Y_{01}S_{21}^*S_{13} + Y_{03}S_{31}^*S_{33} = 0 \tag{10}$$

$$2Y_{01}|S_{13}|^2 + Y_{03}|S_{33}|^2 = Y_{03}. \tag{11}$$

The transducer gain is given by

$$G = |S_{31}|^2 \frac{Y_{03}}{Y_{01}}. \tag{12}$$

However, from reciprocity,

$$Y_{01} \cdot S_{13} = Y_{03} \cdot S_{31}. \tag{13}$$

Substituting (13) into (12),

$$G = |S_{13}|^2 \frac{Y_{01}}{Y_{03}}. \tag{14}$$

From (11)

$$G = \tfrac{1}{2}(1 - |S_{33}|^2). \tag{15}$$

By inspection of (15), it is seen that the maximum conversion gain is obtained when the mixer circuit constants are chosen such that $|S_{33}|$ is zero. Under this condition: *the theoretical limit of conversion loss for a double sideband mixer is 3 dB.*

Since $S_{33} \equiv 0$, (10) reduces to

$$Y_{01} \cdot S_{13} \cdot (S_{11}^* + S_{21}^*) = 0. \tag{16}$$

Since $S_{13} \neq 0$

$$S_{11}^*|_{opt} = -S_{21}^*|_{opt}. \tag{17}$$

Equation (9) then reduces to

$$-2|S_{11}|_{opt}^2 + \frac{Y_{03}}{Y_{01}}|S_{31}|_{opt}^2 = 0. \tag{18}$$

But, since the second term is G_{opt}, which is one half,

$$|S_{11}|_{opt} = \tfrac{1}{2} = |S_{21}|_{opt}.$$

This proves that, in the limit, the lowest conversion loss that can be achieved in a double sideband mixer is 3 dB. The power that is lost is equally divided between conversion to the image and reflection at the signal port (3:1VSWR).

These results have been derived without specifying either the nature of the reactive idler terminations or the ratio of the time the diode is driven into the short-circuit state, to the period of the LO (pulse duty ratio).

This result, therefore, establishes the fundamental limit on performance of a double sideband mixer.

Ultra-Broadband Double Sideband Mixer

In an ultra-broadband double sideband mixer (greater than an octave), the signal and idler spectra overlap. It is not possible to reactively terminate all of the idlers. A meaningful limit on conversion loss can be established by assuming matched loads at all of the idlers. This is most easily done by using an s-matrix model of the diode:

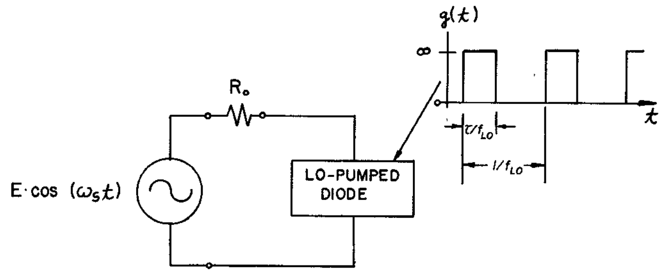

Fig. 3. Circuit used to derive the s parameters of a mixer diode.

$$[b] = [S] \cdot [a]. \tag{19}$$

Truncating the matrix assigns $a_i = 0$ for all ports excluded from the matrix expansion. Therefore, writing a three-port matrix automatically assigns matched loads to the idlers.

To determine the s parameters, the circuit of Fig. 3 is used. A small-signal voltage source with a source impedance R_0 is connected to a time-varying conductance. This conductance is driven between an open-circuit state and a short-circuit state by a local oscillator. The ratio of the time the diode is a short circuit relative to the LO period is denoted as τ.

The voltage and current across the diode are given by

$$v = E\cos(\omega_s t) \cdot [1 - f(t)] \tag{20}$$

$$i = \frac{E\cos(\omega_s t)}{R_0} \cdot f(t) \tag{21}$$

where $f(t)$ is a train of rectangular pulses with unity magnitude, the period is $1/f_{LO}$, and the width is τ/f_{LO}.

The Fourier expansion of $f(t)$ is

$$F\{f(t)\} = \tau + \frac{2}{\pi} \cdot \sum_{n=1}^{\infty} (-1)^n \frac{\sin(n\pi\tau)}{n} \cdot \cos(n\omega_{LO}t). \tag{22}$$

Substituting (22) into (20) and (21),

$$v = (E \cdot (1 - \tau)) \cdot \cos(\omega_s t) - \frac{2E}{\pi} \cdot \sum_{n=1}^{\infty} (-1)^n \frac{\sin(n\pi\tau)}{n} \cdot \cos(n\omega_{LO}t) \cdot \cos(\omega_s t) \tag{23}$$

$$i = \frac{E\cos(\omega_s t)}{R_0} \cdot \tau + \frac{2E}{\pi R_0} \cdot \sum_{n=1}^{\infty} (-1)^n \frac{\sin(n\pi\tau)}{n} \cdot \cos(n\omega_{LO}t) \cdot \cos(\omega_s t). \tag{24}$$

If one postulates a multiplexer at the diode junction, with each output terminated in R_0, the currents and voltages can be combined to determine scattering parameters of an infinite-port network, where each port corresponds to a different frequency.

Scattering parameters are defined as follows:

$$a_1 = \tfrac{1}{2}(V_1/\sqrt{R_0} + I_1\sqrt{R_0}) \tag{25}$$

$$b_1 = \tfrac{1}{2}(V_1/\sqrt{R_0} - I_1\sqrt{R_0}) \quad (26)$$

$$a_N = \tfrac{1}{2}(V_N/\sqrt{R_0} + I_N\sqrt{R_0}) \quad (27)$$

$$b_N = \tfrac{1}{2}(V_N/\sqrt{R_0} - I_N\sqrt{R_0}). \quad (28)$$

Substituting (23) and (24),

$$a_1 = \frac{1}{2}\cdot\frac{E\cos(\omega_s t)}{\sqrt{R_0}} \quad (29)$$

$$a_N = 0 \quad (30)$$

$$b_1 = \frac{1}{2}\cdot\frac{E\cos(\omega_s t)}{\sqrt{R_0}}\cdot(1-2\tau) \quad (31)$$

$$b_N = -\frac{E}{2\sqrt{R_0}}\cdot\left[\frac{2}{\pi}(-1)^n\frac{\sin(n\pi\tau)}{n}\cos(n\omega_{\mathrm{LO}} \pm \omega_s)t\right]. \quad (32)$$

Since the scattering matrix represents complex wave amplitudes, the frequency terms are not carried, but are understood:

$$S_{11} = \frac{b_1}{a_1} = 1 - 2\tau \quad (33)$$

$$S_{N1} = \frac{b_N}{a_1} = -\frac{2}{\pi}\cdot(-1)^n\cdot\frac{\sin(n\pi\tau)}{n}. \quad (34)$$

A similar analysis can be performed for the excitation at each remaining port. The resultant matrix, carried for the three ports, signal, IF, and image, is given by

$$S = \begin{bmatrix} 1-2\tau & \frac{2}{\pi}\sin(\pi\tau) & -\frac{1}{\pi}\sin(2\pi\tau) \\ \frac{2}{\pi}\sin(\pi\tau) & 1-2\tau & \frac{2}{\pi}\sin(\pi\tau) \\ -\frac{1}{\pi}\sin(2\pi\tau) & \frac{2}{\pi}\sin(\pi\tau) & 1-2\tau \end{bmatrix}. \quad (35)$$

Since this mixer is ultra broadband, and the idlers have been assigned matched loads, R_0, it is assumed that the image is also matched and that the signal source impedance is R_0. The degree of freedom is the "pulse duty ratio" [2], τ.

S_{21} is a maximum for $\tau = 0.5$. At this point, the conversion loss is 3.92 dB. This is the value normally assigned as the theoretical limit for double-balanced mixers [4]. As can be seen, at $\tau = 0.5$, the signal and IF ports self-match. This present result shows that this is a general limit for any resistive mixer where all of the higher order mixing products are presented with matched loads.

Conclusions

This paper has examined the fundamental limits on double sideband mixer conversion loss, under the assumption of a "perfect" diode—one with no parasitics, infinite forward conductance, and zero reverse conductance. As would be expected, results have been obtained which are independent of the absolute values of signal source impedance or IF load impedance, since the perfect mixer diode acts as a transformation network.

The first result demonstrated that a double sideband mixer with reactive idler terminations will exhibit, in the limit, a 3-dB conversion loss, with half of the power lost being converted to the image, and the other half lost in a 3 : 1 mismatch at the signal port. Note that any matching network at the signal port also affects the image, so that the mismatch can only be improved at the expense of increased overall conversion loss. This result was determined strictly on the basis of the network properties of the mixer.

A second limitation was obtained for the case where all of the idlers are matched—a good approximation for a multi-octave mixer. There, *s* parameters were employed, and the pumped conductance waveform was used. This result showed a limit of 3.92 dB, independent of mixer configuration (single-ended, balanced, or double-balanced). This optimum occurs for the diode driven into the forward region for 50 percent of the LO period.

Acknowledgment

The author wishes to thank Dr. H. Okean and S. Okwit for their support and encouragement, and to acknowledge the many stimulating discussions with his colleague, S. Foti. He also thanks the reviewers for many important comments on this paper.

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